NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2914

A METHOD FOR RAPID DETERMINATION OF THE ICING LIMIT OF A BODY IN TERMS OF THE STREAM CONDITIONS

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SUMMARY

The effects of existing frictional heating were analyzed to determine the conditions under which ice formations on aircraft surfaces can be prevented. A method is presented for rapidly determining by means of charts the combination of Mach number, altitude, and stream temperature which will maintain an ice-free surface in an icing cloud. The method can be applied to both subsonic and supersonic flow. The charts presented are for Mach numbers up to 1.8 and pressure altitudes from sea level to 45,000 feet.

INTRODUCTION

As the operational speed of aircraft is increased through the transonic region, the frictional heating available to prevent the formation of ice on the aircraft becomes an important quantity. The operation of missiles and interceptor aircraft is therefore possible under predetermined values of altitude, stream temperature, and Mach number which will preclude icing. The set of flight conditions which provides a surface temperature of 32° F for a particular point on a body traveling in an icing cloud is termed the "icing limit" for that point. The analysis presented in reference 1 relates the frictional heating, evaporation, and heat transfer for this particular condition. This analysis was applied to a symmetrical diamond profile airfoil at zero angle of attack and it was found that the most critical region of the airfoil in terms of icing limit was on the rear surface behind the midchord (shoulder). Although this region is not subject to direct impingement of water droplets, the water film caused by droplets impinging on the front surface runs back over the shoulder and extends the hazard of icing over a large percentage of the airfoil. Icing might therefore occur at any point on a body which is subject to droplet impingement or runback.

Experimental verification of the calculations presented in reference 1 was obtained at the NACA Lewis laboratory on a diamond airfoil of

5.50 inch chord and 6.82 percent thickness for a Mach number of 1.36 and several pressure altitudes. The good agreement between experimental and analytical results provides credence for the analytical method presented therein.

The procedure used for calculating the icing limit in reference 1 can be applied generally to obtain the icing limit for any body provided the flow conditions about the body are known. The charts presented herein are based on the analysis of reference 1 for calculating the icing limit at any point on a body. These charts are similar to that shown in figure 1 of reference 1 but cover an altitude range from sea level to 45,000 feet. Several auxiliary charts are also presented from which the pressure coefficient, velocity ratio, or local Mach number at a particular point on the body can be converted to the parameters utilized in the icing limit charts.

SYMBOLS

The following symbols are used in this report:

C_p pressure coefficient, $\frac{p_l - p_0}{1/2\ \rho_0 V_0^2}$

c_p specific heat of air at constant pressure, Btu/(lb)($^{\circ}$F)

e vapor pressure, lb/sq ft

k_e coefficient of evaporation

k_h coefficient of heat transfer

L latent heat of vaporization, Btu/lb

M Mach number

m_a molecular weight of air

m_e molecular weight of water vapor

p static pressure, lb/sq ft

r recovery factor

T static temperature, $^{\circ}$R

$T_{0,c}$ minimum free-stream static temperature corresponding to ice-free condition on surface as defined in eq. (2), $^{\circ}$R

V velocity, ft/sec

γ ratio of specific heats of air, 1.400

ρ density of air, slugs/cu ft

Subscripts:

0 free-stream static conditions

1 local conditions at edge of boundary layer

s surface

ANALYSIS

The generalized heat balance in an icing cloud for an insulated body heated only by frictional effects is given as follows:

(1) Heat due to the frictional, or viscous, effects

plus

(2) Heat due to the kinetic energy of the water droplets

plus

(3) Heat of fusion

equal

(4) Heat lost by convection

plus

(5) Heat for evaporation of water

plus

(6) Heat required to raise temperature of water droplets from stream temperature to surface temperature.

Hardy's relation (ref. 2) is obtained by equating the frictional term (1) to the sum of the convective term (4) and the evaporative term (5). The heat of fusion term (3) is equal to zero if an ice-free surface is defined as being fully wet at 32° F with no ice particles in the surface water film. In addition, terms (2) and (6) are of the same order of magnitude and tend to offset each other for calculations of the icing limit. Both increase in value with increasing Mach number; large values of either term are found only at large values of liquid-water content and flight speed. Such a combination is highly unlikely in actual flight since high speeds usually occur at high altitudes, and high altitudes imply small liquid-water contents with the exception of flight in cumulus clouds of great vertical extent.

The results of reference 1 indicate that the relation of Hardy (ref. 2) applies in particular to a surface barely wetted by a very thin film, but is probably nearly correct for the whole range of speeds and liquid-water contents of practical interest for high-speed flight.

The relation of Hardy is given in reference 1 as

$$T_s = T_1\left[1 + \frac{r(\gamma-1)}{2} M_1^2\right] - \frac{k_e m_e L}{k_h m_a c_p}\left(\frac{e_s}{p_1 - e_s} - \frac{e_1}{p_1 - e_1}\right) \tag{1}$$

Because interest in this problem is fixed on a definition of flight circumstances that provides local surface temperatures of 32° F, the terms T_s, L, and e_s are constants. Hardy observed that for the range of temperatures near 32° F the ratio of the evaporation coefficient k_e to the heat-transfer coefficient k_h is very nearly unity. In addition, the air in a cloud is fully saturated at the static or free-stream conditions and the free-stream vapor pressure e_0 is therefore equal to the saturated vapor pressure at the free-stream static temperature. If it is assumed that the flow about the body, outside the boundary layer, is accomplished with no change in phase, that is, no condensation or evaporation, then Dalton's law of partial pressures applies and

$$\frac{e_1}{e_0} = \frac{p_1}{p_0}$$

Equation (1) can therefore be rewritten in the form

$$492 = \frac{T_1\left(1 + \frac{\gamma-1}{2} r M_1^2\right)}{T_{0,c}} T_{0,c} - 2776.5\left[\frac{12.75}{(p_1/p_0)\, p_0 - 12.75} - \frac{e_0}{p_0 - e_0}\right] \tag{2}$$

which is quite convenient for purposes of calculation because $\left(T_1/T_{0,c}\right)\left(1 + \frac{\gamma-1}{2} r M_1^2\right)$ and p_1/p_0 can be readily calculated at any point on a body for shock-free flow from the pressure or velocity distribution, the stream Mach number, and the recovery factor r. When the flow about the body contains a shock, then the parameters $\left(T_1/T_{0,c}\right)\left(1 + r\,\frac{\gamma-1}{2}\,M_1{}^2\right)$ and p_1/p_0 can be obtained from the local Mach number, the stream Mach number, the pressure coefficient, and the recovery factor r.

PREPARATION OF ICING LIMIT AND AUXILIARY CHARTS

Because e_0 is a function of $T_{0,c}$, the solution of equation (2) must be accomplished by trial and error. In an effort to reduce the laboriousness of such calculations a number of charts have been made and are presented in figure 1. For each pressure altitude considered, the free-stream static temperature corresponding to an ice-free condition $T_{0,c}$ is plotted as a function of the parameter $\left(T_1/T_{0,c}\right)\left(1 + \frac{\gamma-1}{2}\,r\,M_1{}^2\right)$ by means of equation (2) for constant values of p_1/p_0. For each set of values of p_1/p_0 and $\left(T_1/T_{0,c}\right)\left(1 + \frac{\gamma-1}{2}\,r\,M_1{}^2\right)$, a free-stream static temperature exists which corresponds to a surface temperature of 32° F and therefore represents the minimum free-stream static temperature for an ice-free surface $T_{0,c}$. The lower limit of the free-stream static temperature was considered to be -40° F, since it has been shown (refs. 3 and 4) that supercooled water droplets are not likely to exist below this temperature.

The ten icing limit charts presented (figs. 1(a) to 1(j)) were calculated for pressure altitudes from sea level (fig. 1(a)) to 45,000 feet (fig. 1(j)) in 5000 foot increments. The use of the icing limit charts requires the knowledge of the parameters $\left(T_1/T_{0,c}\right)\left(1 + \frac{\gamma-1}{2}\,r\,M_1{}^2\right)$ and p_1/p_0. For shock-free flow both these parameters can be readily calculated from the pressure coefficient C_p or the ratio of local to free-stream velocity V_1/V_0 and the stream Mach number. For most subsonic airfoils either the experimental pressure coefficient or the calculated velocity ratio is known. A variety of methods is available in the literature for determining local values of C_p or V_1/V_0 for almost any body in shock-free flow (refs. 5 to 9).

In the transonic-supersonic regime, wherein a shock normally exists in the flow field, the flow field about a body is not defined by the pressure coefficient and stream Mach number alone and an additional parameter such as shock wave angle or local Mach number is required.

Most of the results in the literature are presented in terms of local pressure coefficient and local Mach number for constant values of free-stream Mach number (ref. 10).

The pressure coefficient may be related to the pressure ratio and the stream Mach number by the compressible flow relations for a perfect gas without the assumption of isentropic flow as follows:

$$\left.\begin{aligned} C_p &= \frac{p_1 - p_0}{\frac{1}{2}\rho_0 V_0^2} = \frac{2(p_1 - p_0)}{\gamma p_0 M_0^2} = \frac{2}{\gamma M_0^2}\left(\frac{p_1}{p_0} - 1\right) \\ \frac{p_1}{p_0} &= 1 + \frac{\gamma}{2} M_0^2 C_p = 1 + 0.7 M_0^2 C_p \end{aligned}\right\} \tag{3}$$

Hence this result applies to both shocked and shock-free flows.

The velocity ratio may be related to the pressure ratio and the stream Mach number in the following manner for shock-free flow:

$$\left(\frac{V_1}{V_0}\right)^2 = \frac{\rho_1 V_1^2}{\rho_0 V_0^2}\frac{\rho_0}{\rho_1} = \frac{\rho_0 p_1 M_1^2}{\rho_1 p_0 M_0^2} = \left(\frac{p_1}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\left(\frac{M_1}{M_0}\right)^2$$

The static pressure - Mach number relation for isentropic flow is given by

$$\left(\frac{p_1}{p_0}\right)^{\frac{\gamma-1}{\gamma}} = \frac{1 + \frac{\gamma-1}{2} M_0^2}{1 + \frac{\gamma-1}{2} M_1^2}$$

Solving for M_1^2 gives

$$M_1^2 = \left[\left(\frac{p_1}{p_0}\right)^{-\frac{\gamma-1}{\gamma}}\left(1 + \frac{\gamma-1}{2} M_0^2\right) - 1\right]\frac{2}{\gamma-1}$$

Therefore

$$\left(\frac{V_1}{V_0}\right)^2 = \left(\frac{p_1}{p_0}\right)^{\frac{\gamma-1}{\gamma}} \frac{1}{M_0^2}\left[\left(\frac{p_1}{p_0}\right)^{-\frac{\gamma-1}{\gamma}}\left(1 + \frac{\gamma-1}{2} M_0^2\right) - 1\right]\frac{2}{\gamma-1}$$

$$= \frac{2}{(\gamma-1)\ M_0^2}\left[1 + \frac{\gamma-1}{2} M_0^2 - \left(\frac{p_1}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

$$\frac{V_1}{V_0} = \frac{2.236}{M_0}\sqrt{1 + 0.2\ M_0^2 - \left(\frac{p_1}{p_0}\right)^{0.286}} \tag{4}$$

The parameter $\left(T_1/T_{0,c}\right)\left(1 + \frac{\gamma-1}{2} r\ M_1^2\right)$ is related to p_1/p_0, the stream Mach number, and the recovery factor in the following way for shock-free flow:

$$\frac{T_1}{T_{0,c}}\left(1 + \frac{\gamma-1}{2} r\ M_1^2\right) = \left(\frac{p_1}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\left(1 + \frac{\gamma-1}{2} r\ M_1^2\right)$$

but

$$1 + \frac{\gamma-1}{2} M_1^2 = \left(\frac{p_1}{p_0}\right)^{-\frac{\gamma-1}{\gamma}}\left(1 + \frac{\gamma-1}{2} M_0^2\right)$$

Multiplying both sides by r, adding unity to each side, and rearranging terms yield

$$1 + r\frac{\gamma-1}{2} M_1^2 = \left(\frac{p_1}{p_0}\right)^{-\frac{\gamma-1}{\gamma}}\left(1 + \frac{\gamma-1}{2} M_0^2\right) r + (1 - r)$$

Therefore, by substitution,

$$\frac{T_1}{T_{0,c}}\left(1 + \frac{\gamma-1}{2} r\ M_1^2\right) = \left(1 + \frac{\gamma-1}{2} M_0^2\right) r + (1 - r)\left(\frac{p_1}{p_0}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= \left(1 + 0.2\ M_0^2\right) r + (1 - r)\left(\frac{p_1}{p_0}\right)^{0.286} \tag{5}$$

For shocked flows the pressure ratio can be obtained by equation (3), but the parameter $\left(T_1/T_{0,c}\right)\left(1 + \frac{\gamma-1}{2} r M_1^2\right)$ can no longer be related to the pressure ratio and stream Mach number as for shock-free flows (eq. (5)).

The parameter $\left(T_1/T_{0,c}\right)\left(1 + \frac{\gamma-1}{2} r M_1^2\right)$ can, however, be related quite simply to the local and free-stream Mach numbers. The temperature ratio T_1/T_0 is given in terms of Mach number by the energy relations which apply to both shocked and shock-free flows as follows:

$$\frac{T_1}{T_0} = \frac{1 + \frac{\gamma-1}{2} M_0^2}{1 + \frac{\gamma-1}{2} M_1^2}$$

therefore

$$\frac{T_1}{T_{0,c}}\left(1 + \frac{\gamma-1}{2} r M_1^2\right) = \frac{1 + \frac{\gamma-1}{2} M_0^2}{1 + \frac{\gamma-1}{2} M_1^2}\left(1 + \frac{\gamma-1}{2} r M_1^2\right) \tag{6}$$

The pressure ratio p_1/p_0 can be readily obtained by use of equation (3) if M_0 and C_p are known. The term $M_0^2 C_p$ can be calculated from experimental pressure distributions which are usually presented for each value of M_0 in terms of C_p.

Theoretical velocity distributions for many bodies are given in terms of the velocity ratio or the square of the velocity ratio. The conversion from velocity ratio to pressure ratio (eq. (4)) is shown in figure 2, where the pressure ratio is plotted as a function of velocity ratio for constant values of the stream Mach number.

Figure 3 shows the conversion from pressure ratio to the parameter $\left(T_1/T_{0,c}\right)\left(1 + \frac{\gamma-1}{2} r M_0^2\right)$ as given by equation (5) for shock-free flows. The parameter $\left(T_1/T_{0,c}\right)\left(1 + \frac{\gamma-1}{2} r M_0^2\right)$ is plotted as a function of the stream Mach number M_0 for constant values of the pressure ratio p_1/p_0. Figures 3(a), 3(b), and 3(c) are given for values of the recovery factor r of 0.80, 0.85, and 0.90, respectively.

The relation given by equation (6) is shown plotted in figure 4 where the parameter $\left(T_1/T_{0,c}\right)\left(1 + \frac{\gamma-1}{2} r M_1^2\right)$ is plotted as a function of the stream Mach number for constant values of the local to free-stream Mach number ratio M_1/M_0. Figures 4(a), 4(b), and 4(c) are given for values of the recovery factor of 0.80, 0.85, and 0.90, respectively.

Discussion and Illustrative Examples

Most aircraft components which are exposed to icing are of airfoil or streamlined shape. For this general class of bodies most of the cloud water droplets impinge near the leading edge or nose region. The icing hazard, however, is not limited to the region of direct impingement, since if the impinged cloud droplets do not freeze (because of frictional heating) they create a surface water film which flows back from the impingement area. The whole region wetted by the film must therefore be considered as a region of possible icing hazard.

The critical point on an airfoil or similar body (i.e., the point where the largest value of $T_{0,c}$ is required for a particular value of stream Mach number) occurs at the minimum pressure point because the decreased pressure enhances the evaporation and results in a reduced surface temperature. If impingement occurs only near the leading edge or nose region, the evaporation in the region ahead of the point of minimum pressure may be sufficient to remove all the water film and, in such a case, the results obtained from charts 1(a) to 1(j) would not apply. The actual critical point might therefore occur forward of the minimum pressure point; therefore, values of $T_{0,c}$ based on minimum pressure at a particular Mach number would be greater than actually required.

The use of charts 1(a) to 1(j) can be shown most readily by means of several examples. The following example illustrates typical results in the subsonic speed range.

Example I. - Calculation is made of the free-stream static temperature required for an ice-free surface as a function of Mach number for an NACA 65-206 airfoil at 15,000 feet pressure altitude and 1° angle of attack.

The velocity ratio V_1/V_0 can be found for this airfoil by the methods and results presented in reference 5. The maximum velocity ratio (minimum pressure ratio) for this airfoil was determined to be $V_1/V_0 = 1.139$ and occurs at the 45-percent chord station on the upper surface. Assuming a value of the recovery factor of 0.85 and that the surface is wet at this point determines $T_{0,c}$ in the following way:

Free-stream Mach number, M_0	Static-pressure ratio, p_1/p_0 (fig. 2(b) and V_1/V_0)	Parameter $\frac{T_1}{T_{0,c}}\left(1 + \frac{\gamma-1}{2} r M_1^2\right)$ [fig. 3(b), $\frac{p_1}{p_0}$, M_0]	Minimum free-stream static temperature, $T_{0,c}$, ^{o}R [fig. 1(d)]
0.400	0.970	1.026	486.8
.500	.950	1.041	483.8
.600	.929	1.059	480.0
.700	.902	1.079	475.6

The values of $T_{0,c}$ obviously decrease with increasing Mach number as would be expected. The results of example 1 show, however, that even at a Mach number of 0.7, protection is provided at stream temperatures greater than 16° F.

Example 2. - This example is presented to show typical calculations and results in the transonic speed range. Determination is made of the relation between stream Mach number and stream static temperature which will provide an ice-free surface at the midchord of an 8.8 percent thick circular arc airfoil at zero angle of attack for altitudes of 10,000, 25,000, and 40,000 feet. The recovery factor r is 0.90.

The values of pressure coefficient C_p and local Mach number M_1 can be obtained from reference 10 for stream Mach numbers of 0.848 to 1.500 and are listed as follows:

M_0 (ref. 10)	M_1 (ref. 10) at mid-chord	C_p (ref. 10) at mid-chord	$0.7 M_0^2 C_p$ (calculated)	$\frac{p_1}{p_0}$ (eq. (3))	$\frac{M_1}{M_0}$ (calculated)	$\frac{T_1}{T_{0,c}}(A)$[1] (fig. 4(c), M_0, $\frac{M_1}{M_0}$)	$T_{0,c}$, ^{o}R Altitude, ft 10,000 (fig. 1(c))	25,000 (fig. 1(f))	40,000 (fig. 1(i))
0.848	1.023	-0.355	-0.179	0.820	1.206	1.123	460.0	472.5	484.0
.935	1.140	-.330	-.202	.798	1.219	1.150	452.0	466.5	481.5
1.110	1.160	-.045	-.039	.961	1.045	1.220	423.5	440.0	464.0
1.160	1.183	-.028	-.026	.976	1.020	1.241	-----	432.5	459.0
1.200	1.195	.010	.010	.990	.996	1.260	-----	425.5	454.0
1.250	1.225	.025	.027	1.027	.980	1.282	-----	-----	447.5
1.350	1.306	.028	.036	1.036	.967	1.331	-----	-----	435.0

[1] $A = 1 + \frac{\gamma-1}{2} r M_1^2$.

The results of example 2 are shown plotted in figure 5, where the stream static temperature corresponding to an ice-free condition is plotted as a function of the flight Mach number for the three altitudes considered. The effect of altitude is clearly illustrated. The free-stream static temperatures required for protection are higher at the higher pressure altitudes, as would be expected; since, if other conditions are equal, the decreased pressure enhances the evaporation rate.

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National Advisory Committee for Aeronautics
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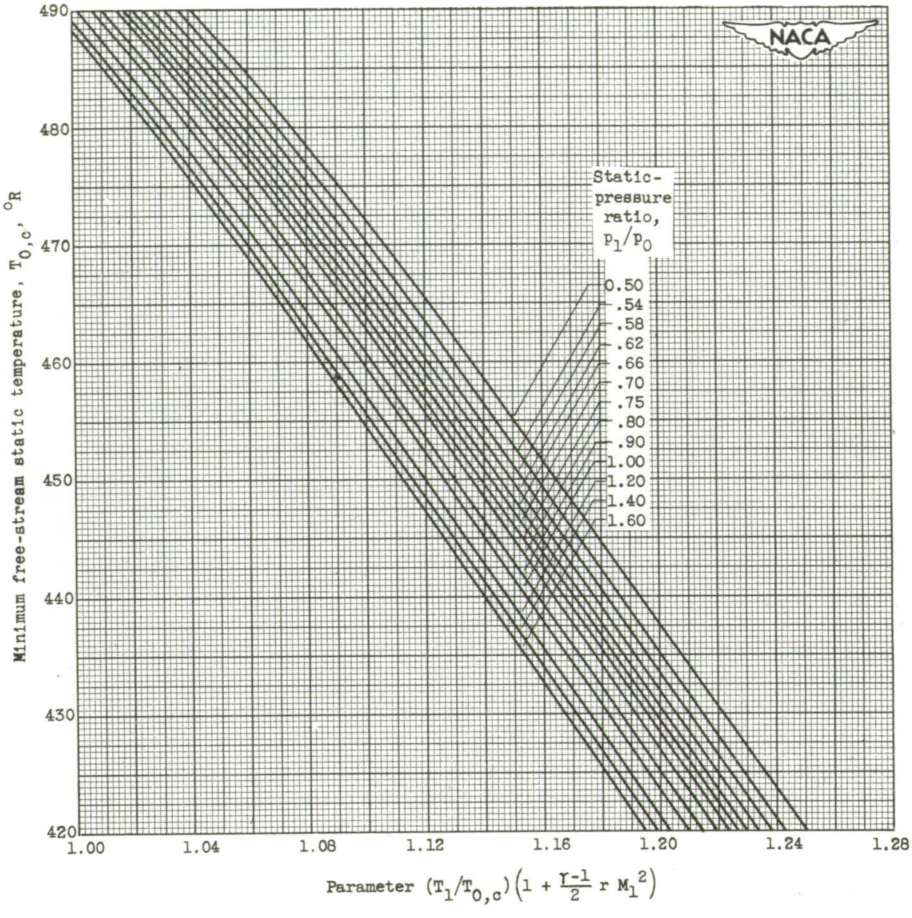
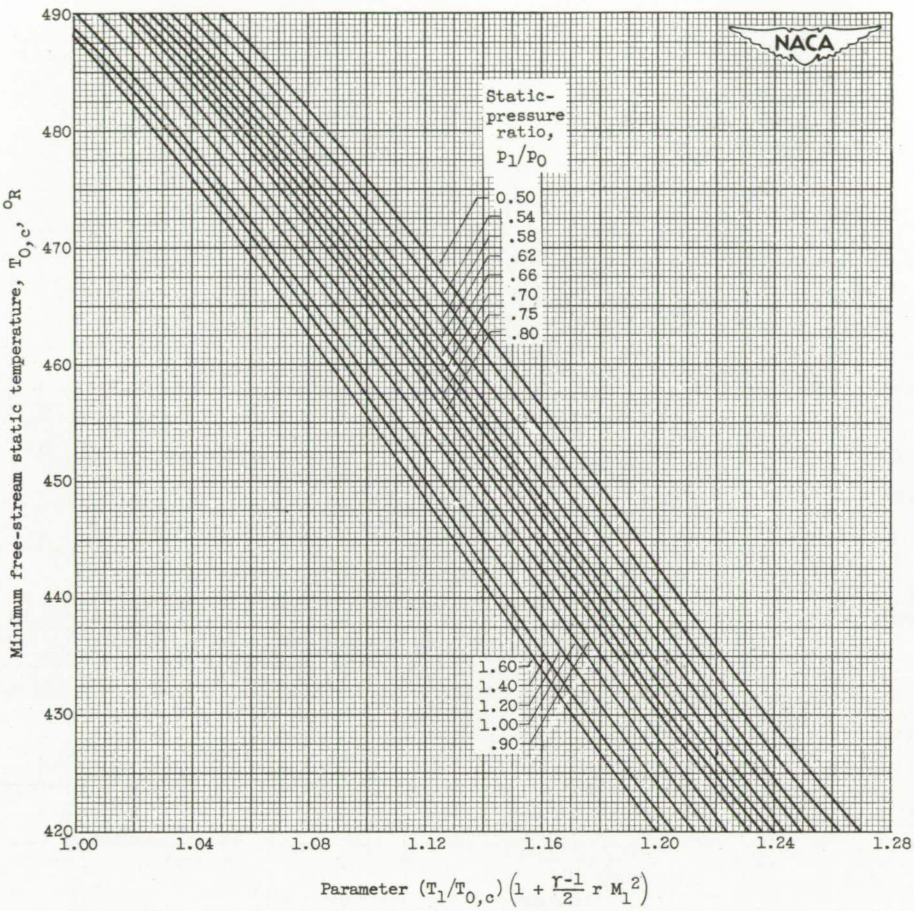


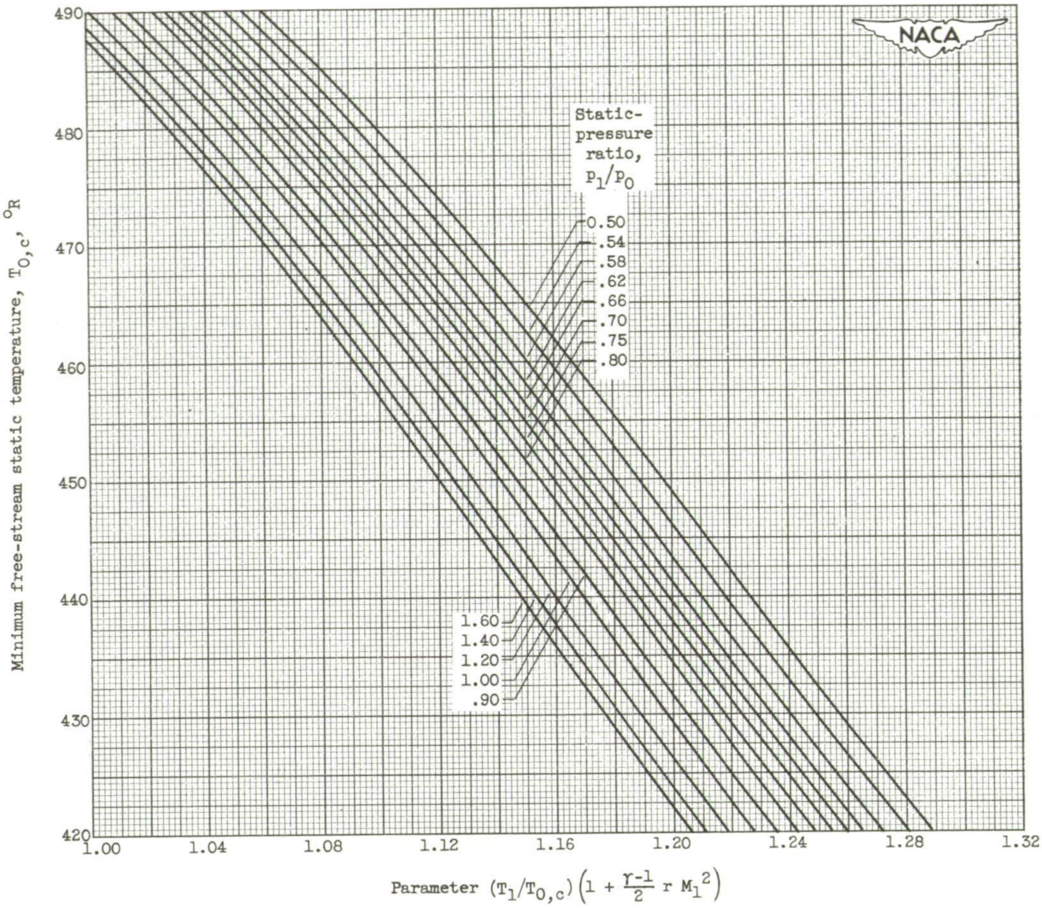
(a) Pressure altitude, sea level.

Figure 1. - Variation of minimum free-stream static temperature $T_{0,c}$ corresponding to the ice-free condition with parameter $(T_1/T_{0,c})\left(1 + \frac{\gamma-1}{2} r M_1^2\right)$ for various pressure ratios (eq. (2)).



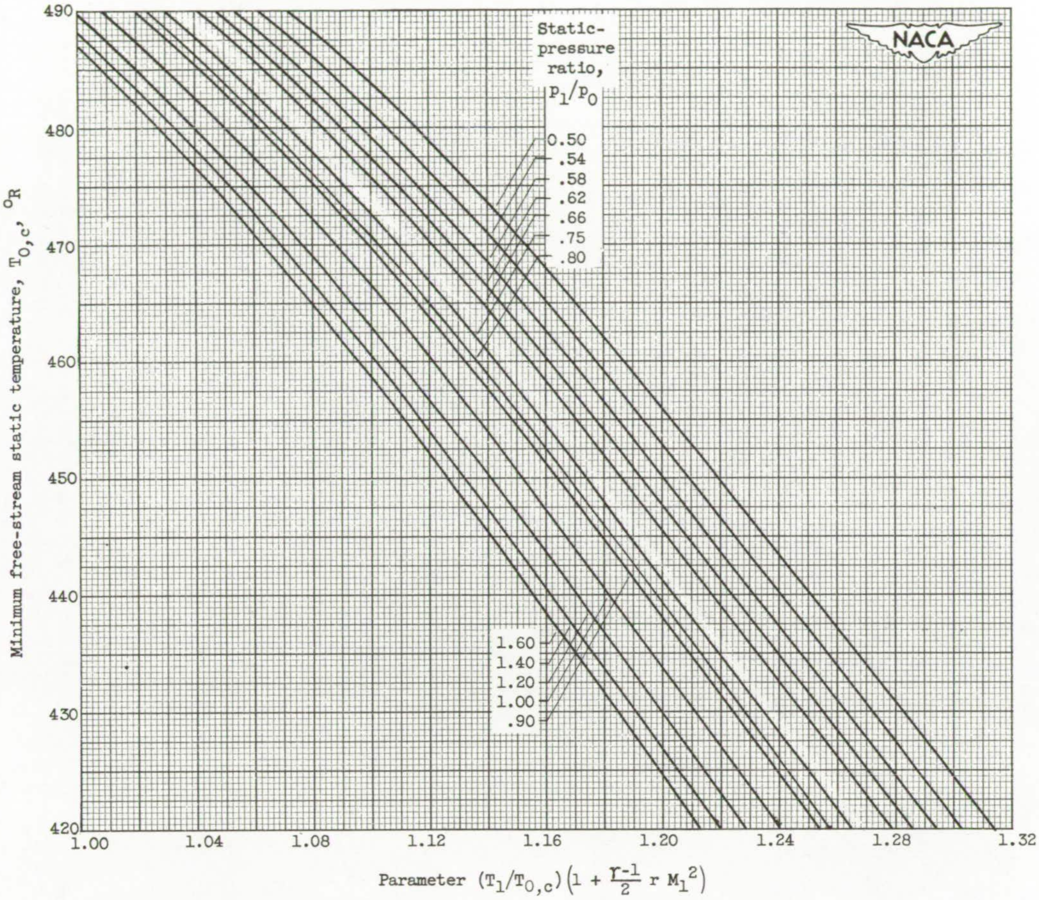
(b) Pressure altitude, 5000 feet.

Figure 1. - Continued. Variation of minimum free-stream static temperature $T_{0,c}$ corresponding to the ice-free condition with parameter $(T_1/T_{0,c})\left(1 + \frac{\gamma-1}{2} r M_1^2\right)$ for various pressure ratios (eq. (2)).



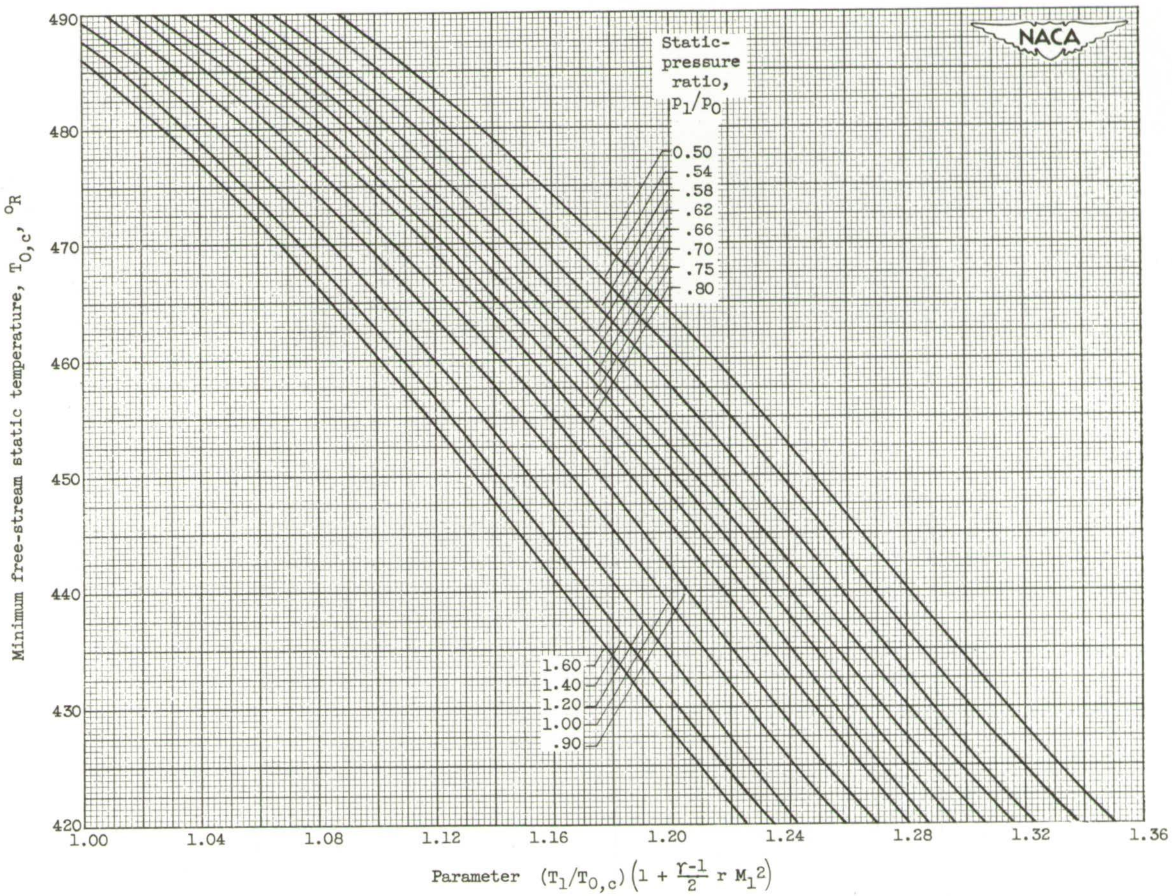
(c) Pressure altitude, 10,000 feet.

Figure 1. - Continued. Variation of minimum free-stream static temperature $T_{0,c}$ corresponding to the ice-free condition with parameter $(T_1/T_{0,c})\left(1 + \frac{\gamma-1}{2} r M_1^2\right)$ for various pressure ratios (eq. (2)).



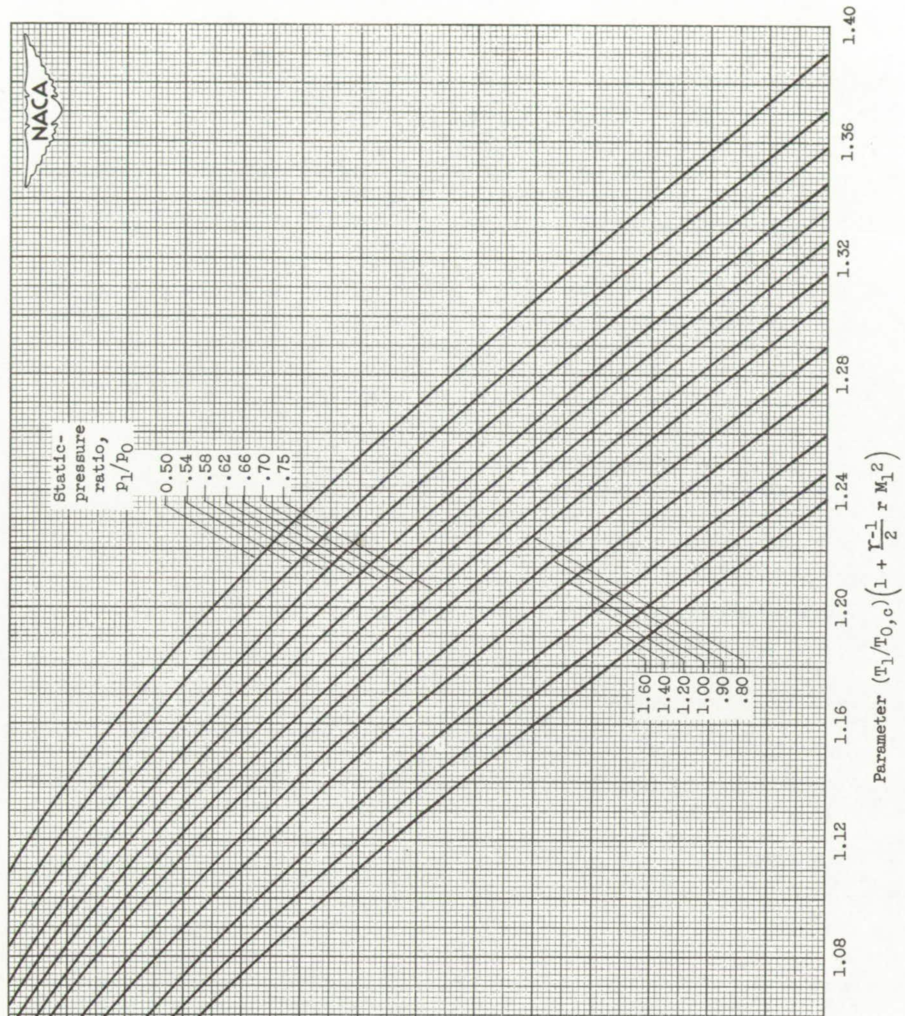
(d) Pressure altitude, 15,000 feet.

Figure 1. - Continued. Variation of minimum free-stream static temperature $T_{0,c}$ corresponding to the ice-free condition with parameter $(T_1/T_{0,c})\left(1 + \frac{\gamma-1}{2} r M_1^2\right)$ for various pressure ratios (eq. (2)).



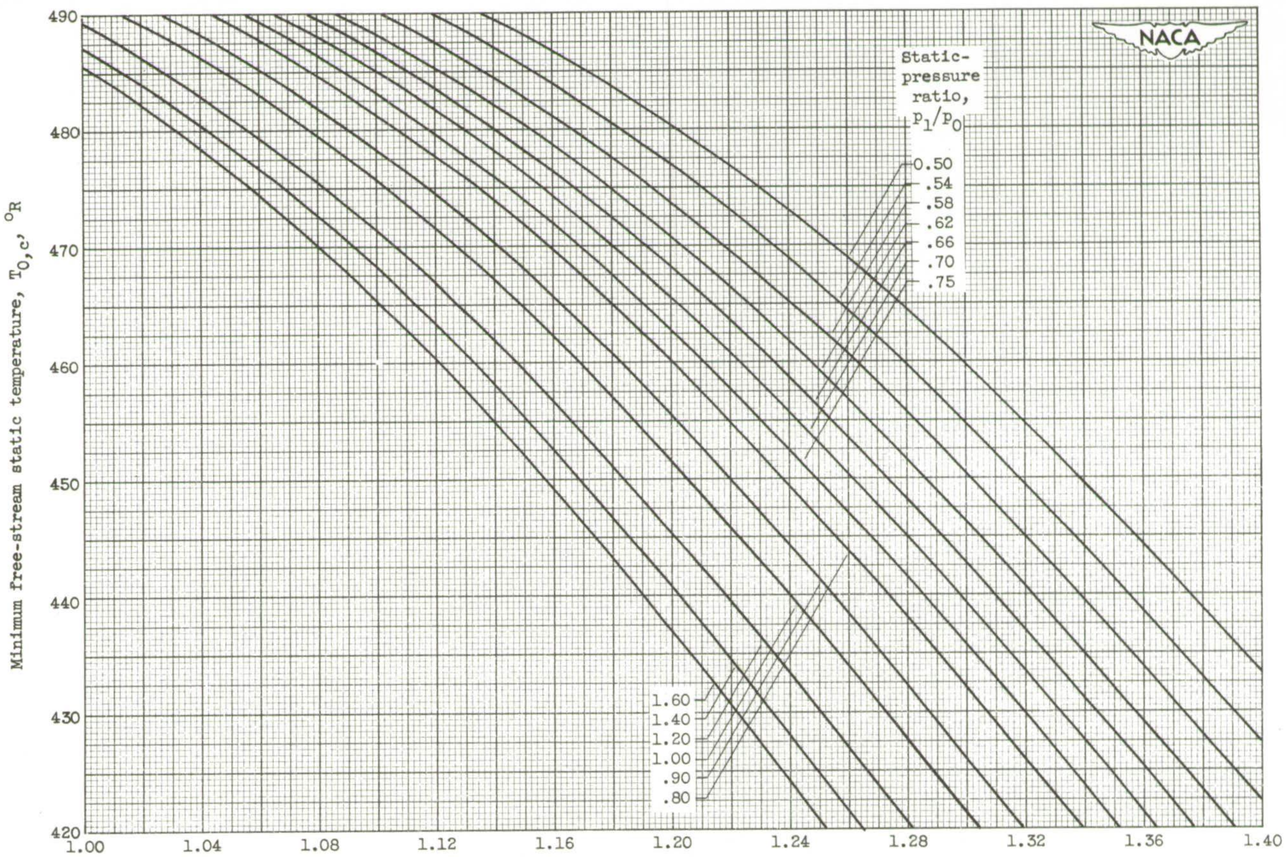
(e) Pressure altitude, 20,000 feet.

Figure 1. - Continued. Variation of minimum free-stream static temperature $T_{0,c}$ corresponding to the ice-free condition with parameter $(T_1/T_{0,c})\left(1 + \frac{\gamma-1}{2} r M_1^2\right)$ for various pressure ratios (eq. (2)).



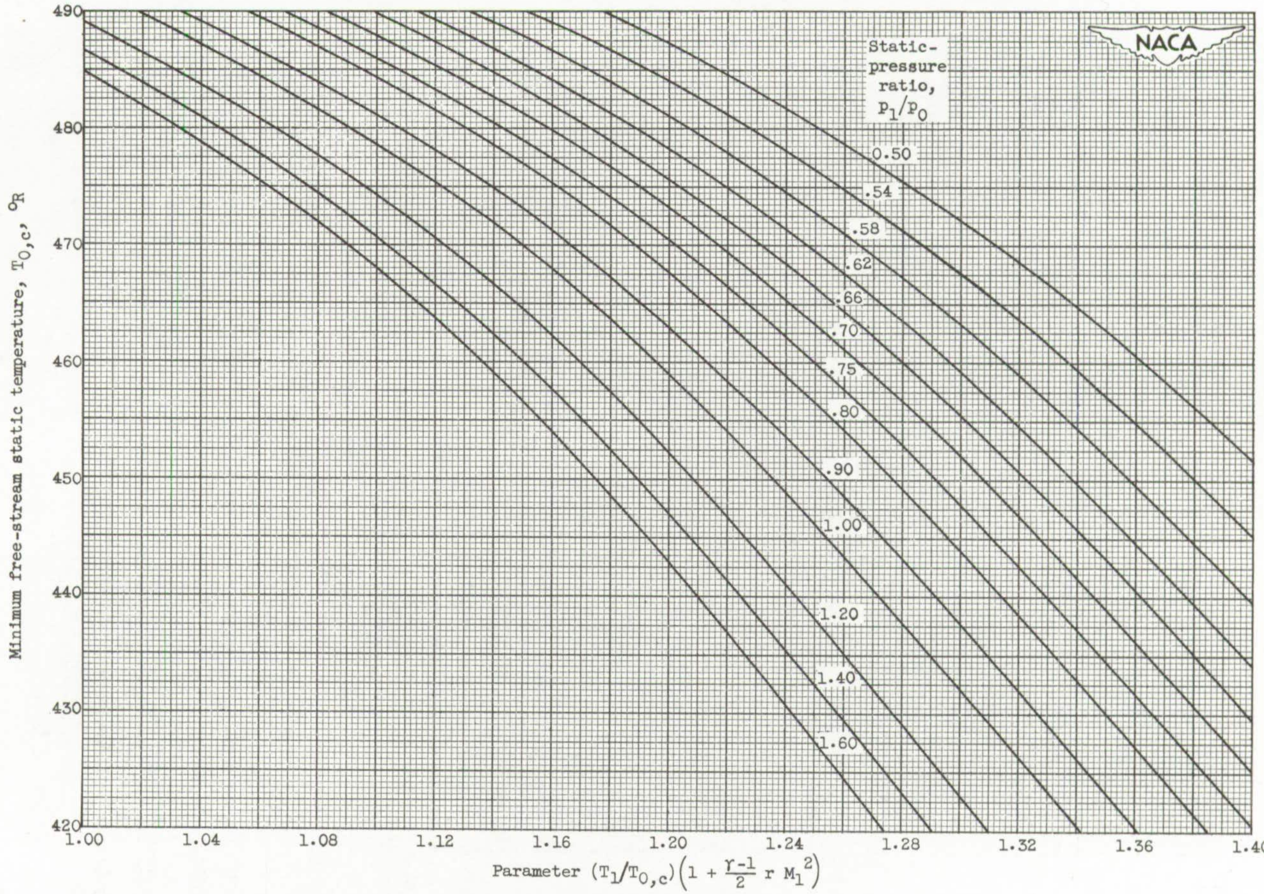
(f) Pressure altitude, 25,000 feet.

Figure 1. - Continued. Variation of minimum free-stream static temperature $T_{0,c}$ corresponding to the ice-free condition with parameter $(T_1/T_{0,c})\left(1 + \frac{\gamma-1}{2} r M_1^2\right)$ for various pressure ratios (eq. (2)).



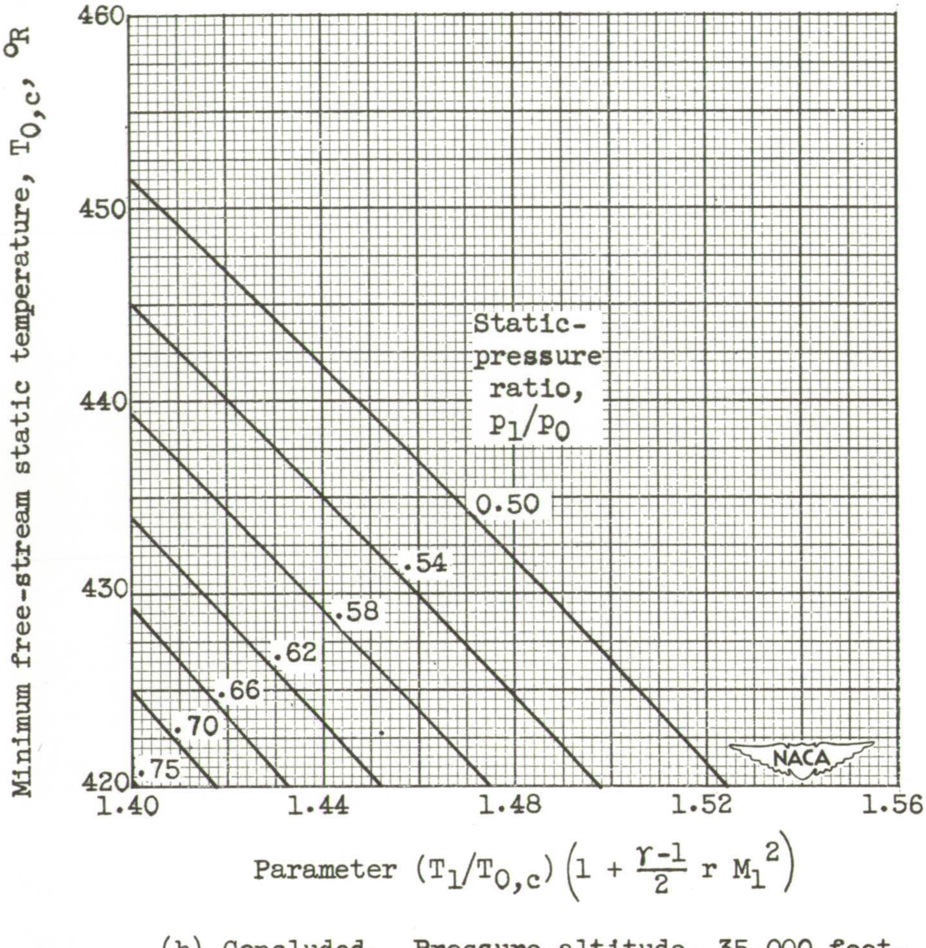
(g) Pressure altitude, 30,000 feet.

Figure 1. - Continued. Variation of minimum free-stream static temperature $T_{0,c}$ corresponding to the ice-free condition with parameter $(T_1/T_{0,c})\left(1 + \frac{\gamma-1}{2} r M_1^2\right)$ for various pressure ratios (eq. (2)).



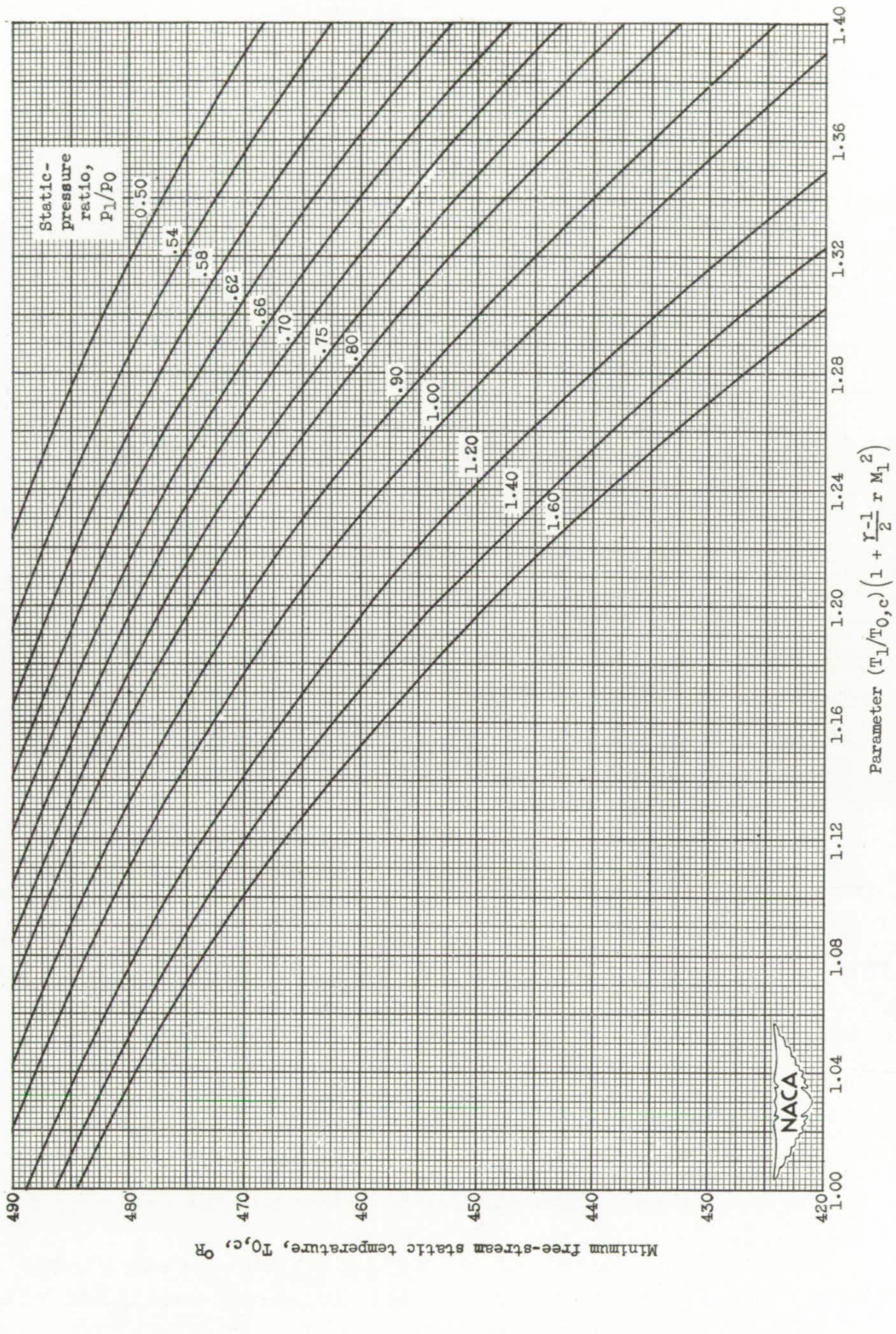
(h) Pressure altitude, 35,000 feet.

Figure 1. - Continued. Variation of minimum free-stream static temperature $T_{0,c}$ corresponding to the ice-free condition with parameter $(T_1/T_{0,c})\left(1 + \frac{\gamma-1}{2} r M_1^2\right)$ for various pressure ratios (eq. (2)).



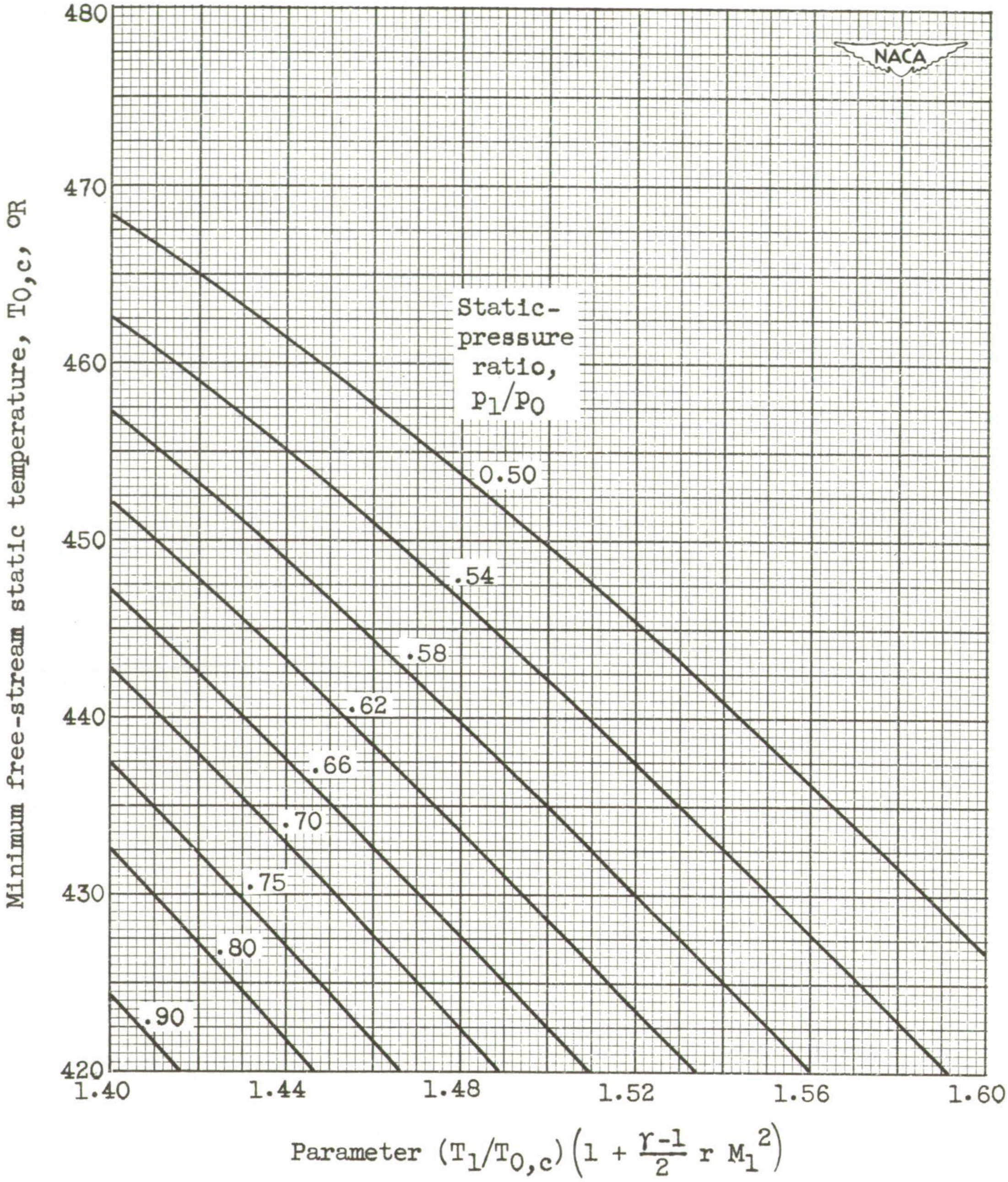
(h) Concluded. Pressure altitude, 35,000 feet.

Figure 1. - Continued. Variation of minimum free-stream static temperature $T_{0,c}$ corresponding to the ice-free condition with parameter $(T_1/T_{0,c})\left(1 + \frac{\gamma-1}{2} r M_1^2\right)$ for various pressure ratios (eq. (2)).



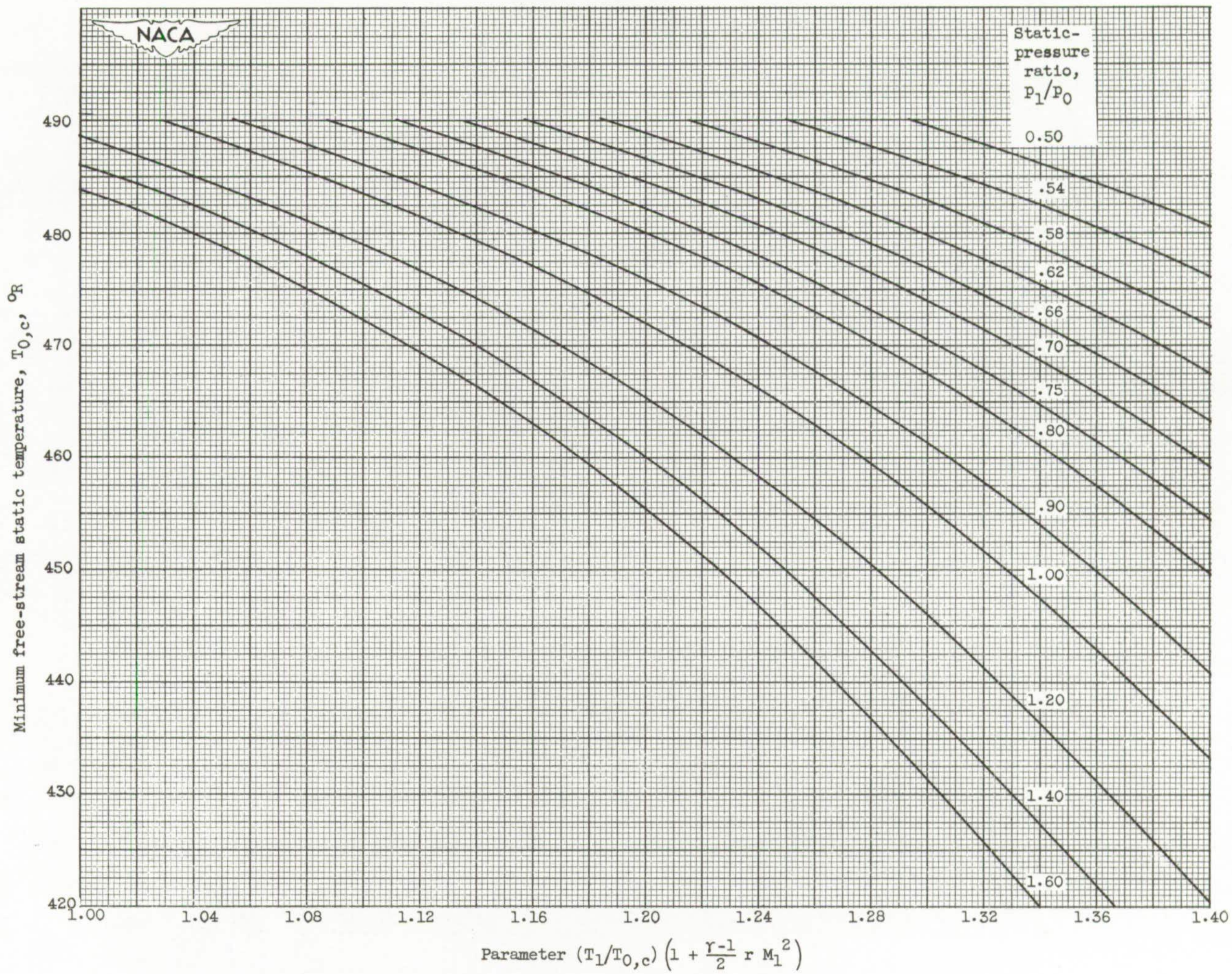
(l) Pressure altitude, 40,000 feet.

Figure 1. - Continued. Variation of minimum free-stream static temperature $T_{0,c}$ corresponding to the ice-free condition with parameter $(T_1/T_{0,c})\left(1 + \frac{\gamma-1}{2} r M_1^2\right)$ for various pressure ratios (eq. (2)).



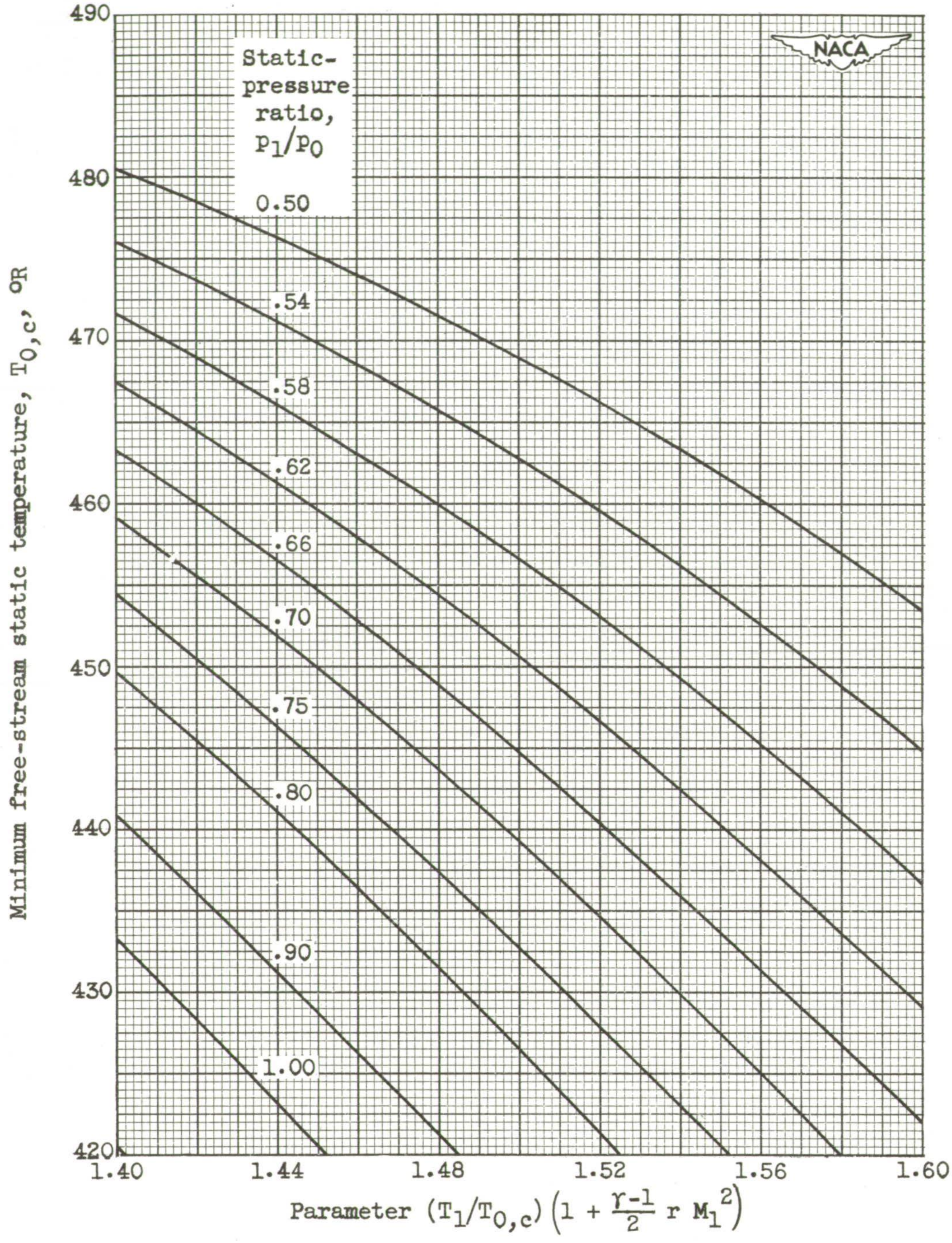
(i) Concluded. Pressure altitude, 40,000 feet.

Figure 1. - Continued. Variation of minimum free-stream static temperature $T_{0,c}$ corresponding to the ice-free condition with parameter $(T_1/T_{0,c})\left(1 + \frac{\gamma-1}{2} r M_1^2\right)$ for various pressure ratios (eq. (2)).



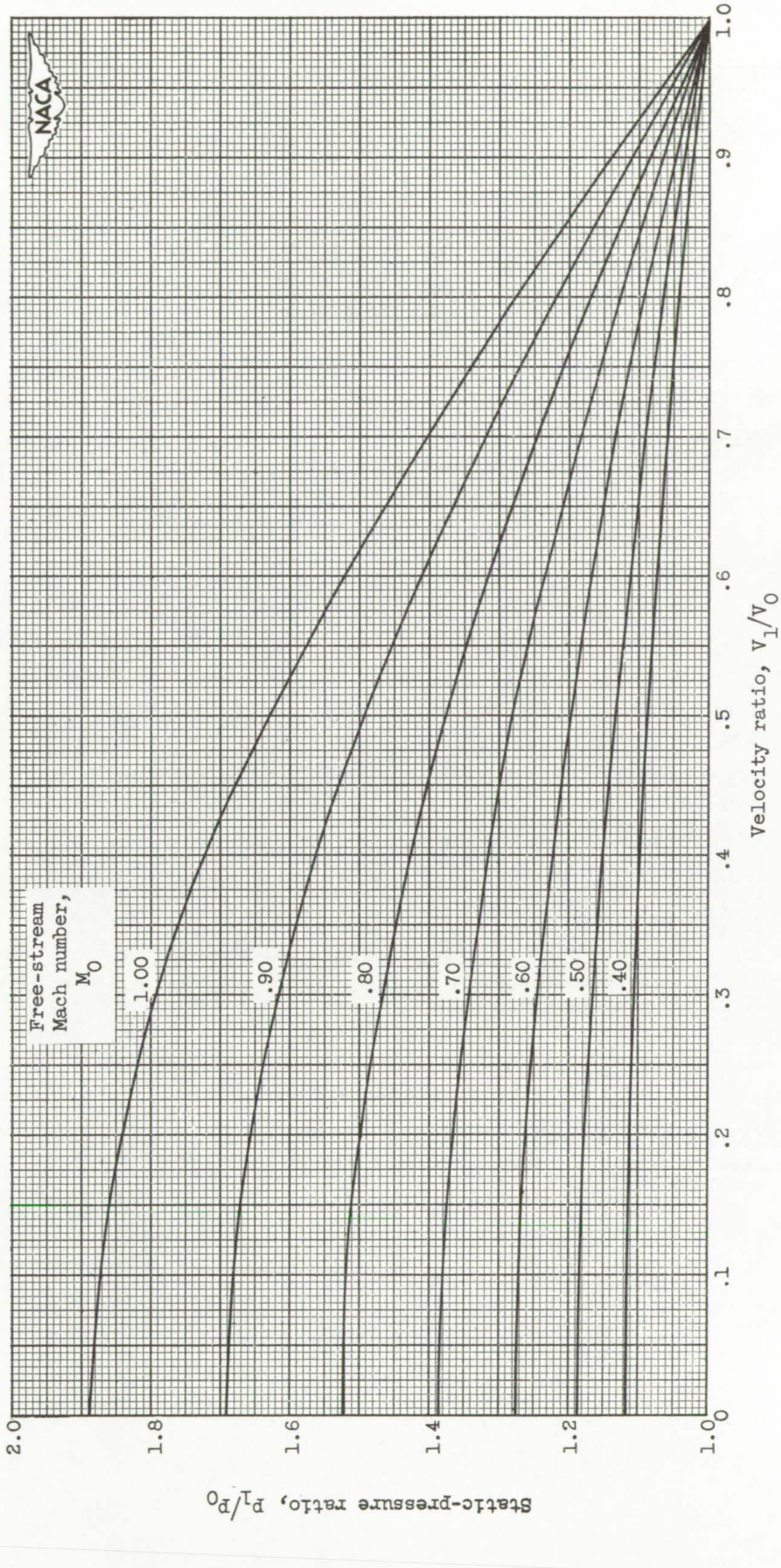
(j) Pressure altitude, 45,000 feet.

Figure 1. - Continued. Variation of minimum free-stream static temperature $T_{0,c}$ corresponding to the ice-free condition with parameter $(T_1/T_{0,c})\left(1 + \frac{\gamma-1}{2} r M_1^2\right)$ for various pressure ratios (eq. (2)).



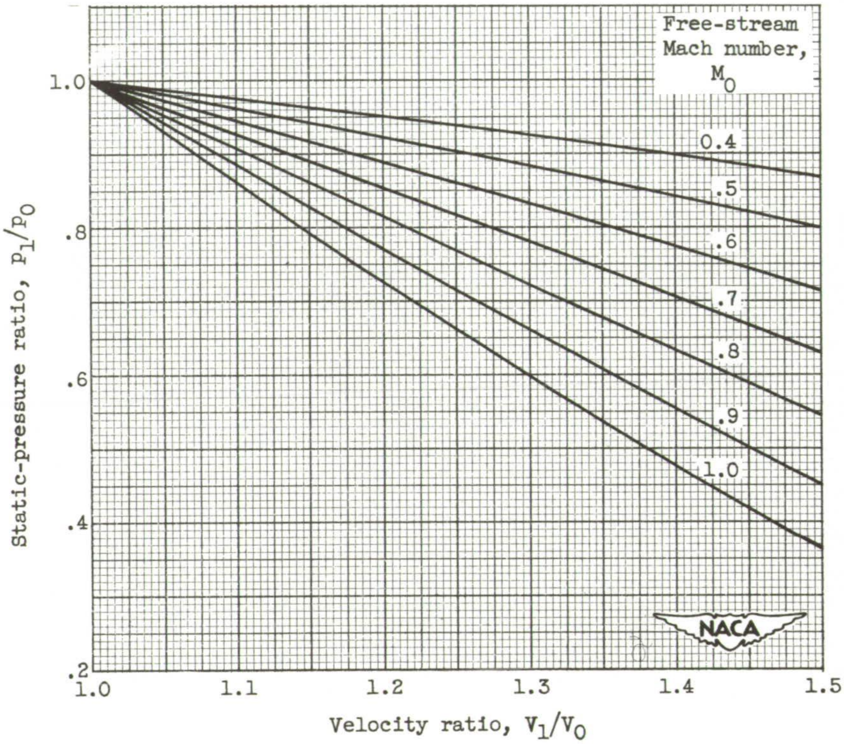
(j) Concluded. Pressure altitude, 45,000 feet.

Figure 1. - Concluded. Variation of minimum free-stream static temperature $T_{0,c}$ corresponding to the ice-free condition with parameter $(T_1/T_{0,c})\left(1 + \frac{\gamma-1}{2} r M_1^2\right)$ for various pressure ratios (eq. (2)).



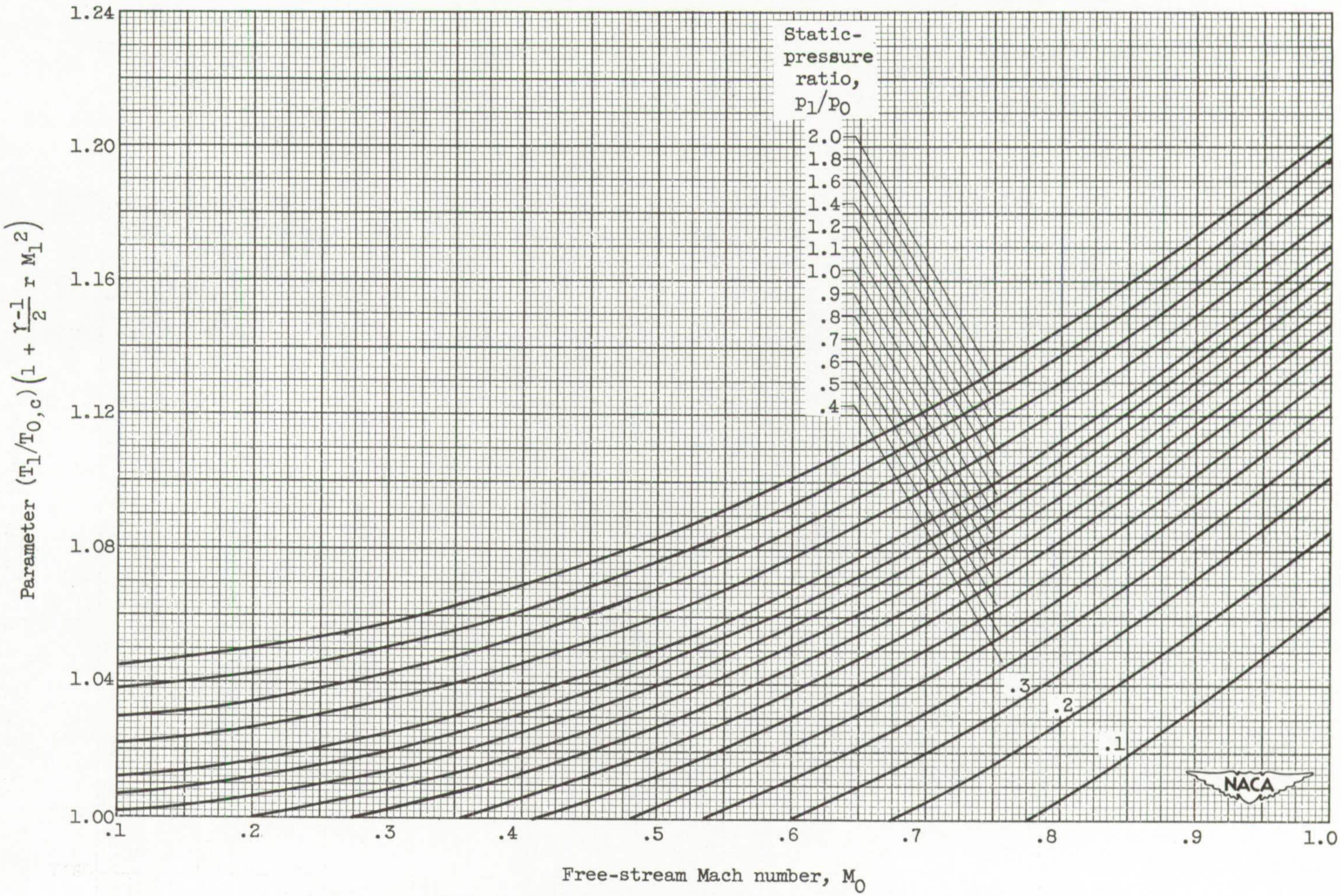
(a) Velocity ratios less than 1.

Figure 2. - Variation of static-pressure ratio with velocity ratio in shock-free flow for various free-stream Mach numbers (eq. (4)).



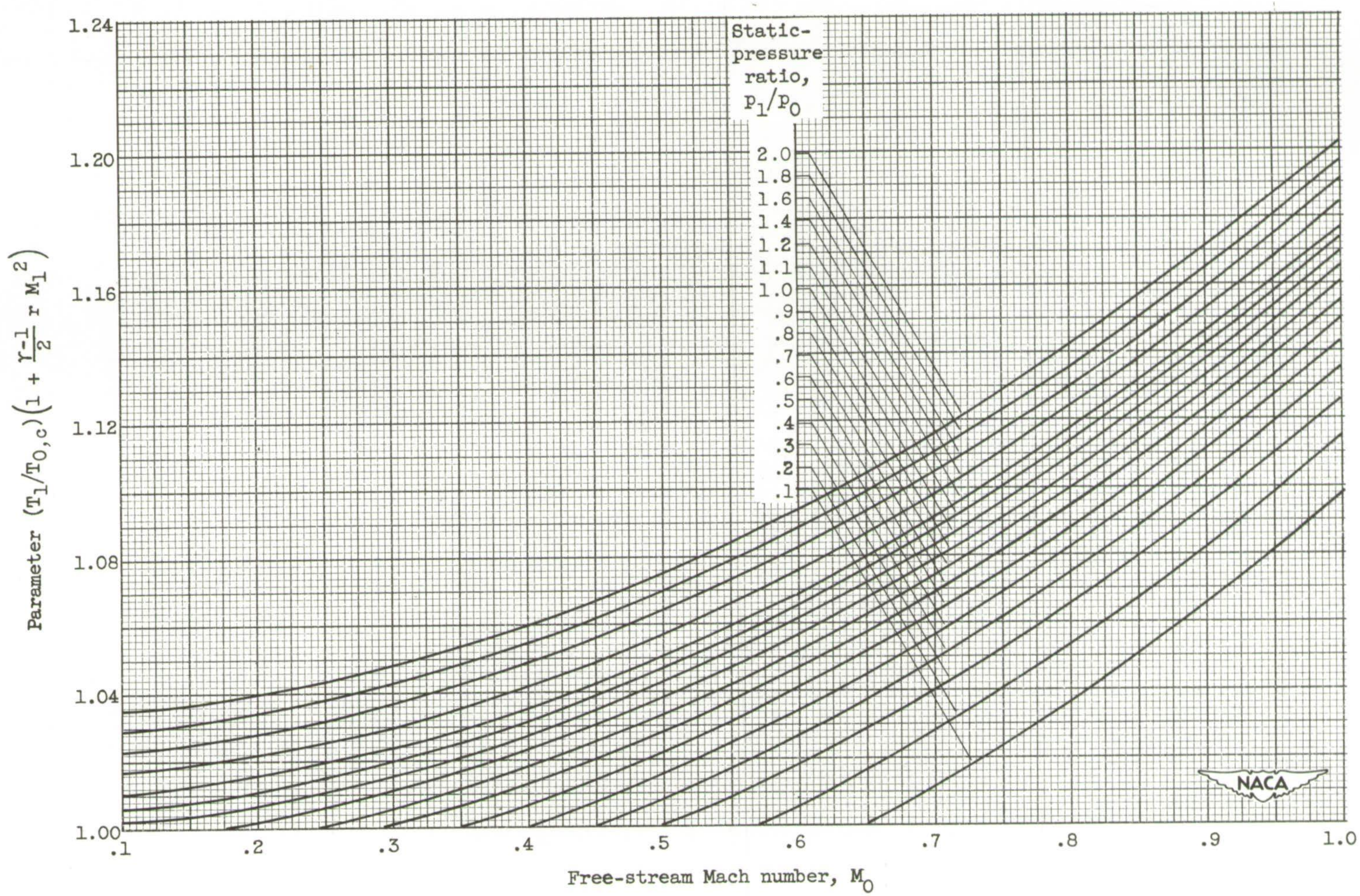
(b) Velocity ratios greater than 1.

Figure 2. - Concluded. Variation of static-pressure ratio with velocity ratio in shock-free flow for various free-stream Mach numbers (eq. (4)).



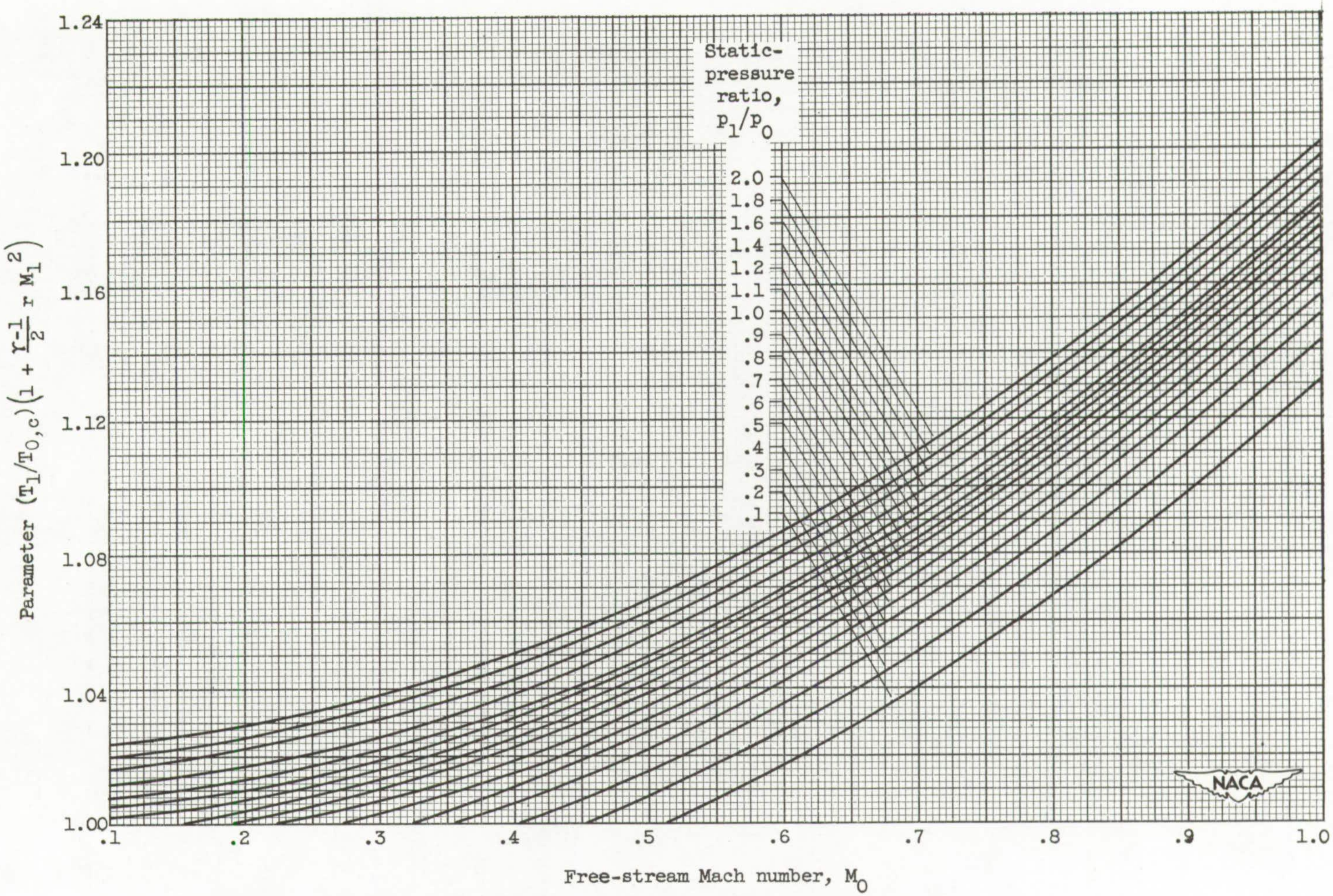
(a) Recovery factor, 0.80.

Figure 3. - Variation of parameter $(T_1/T_{0,c})\left(1 + \frac{\gamma-1}{2} r M_1^2\right)$ with free-stream Mach number in shock-free flow for various static-pressure ratios (eq. (5)).



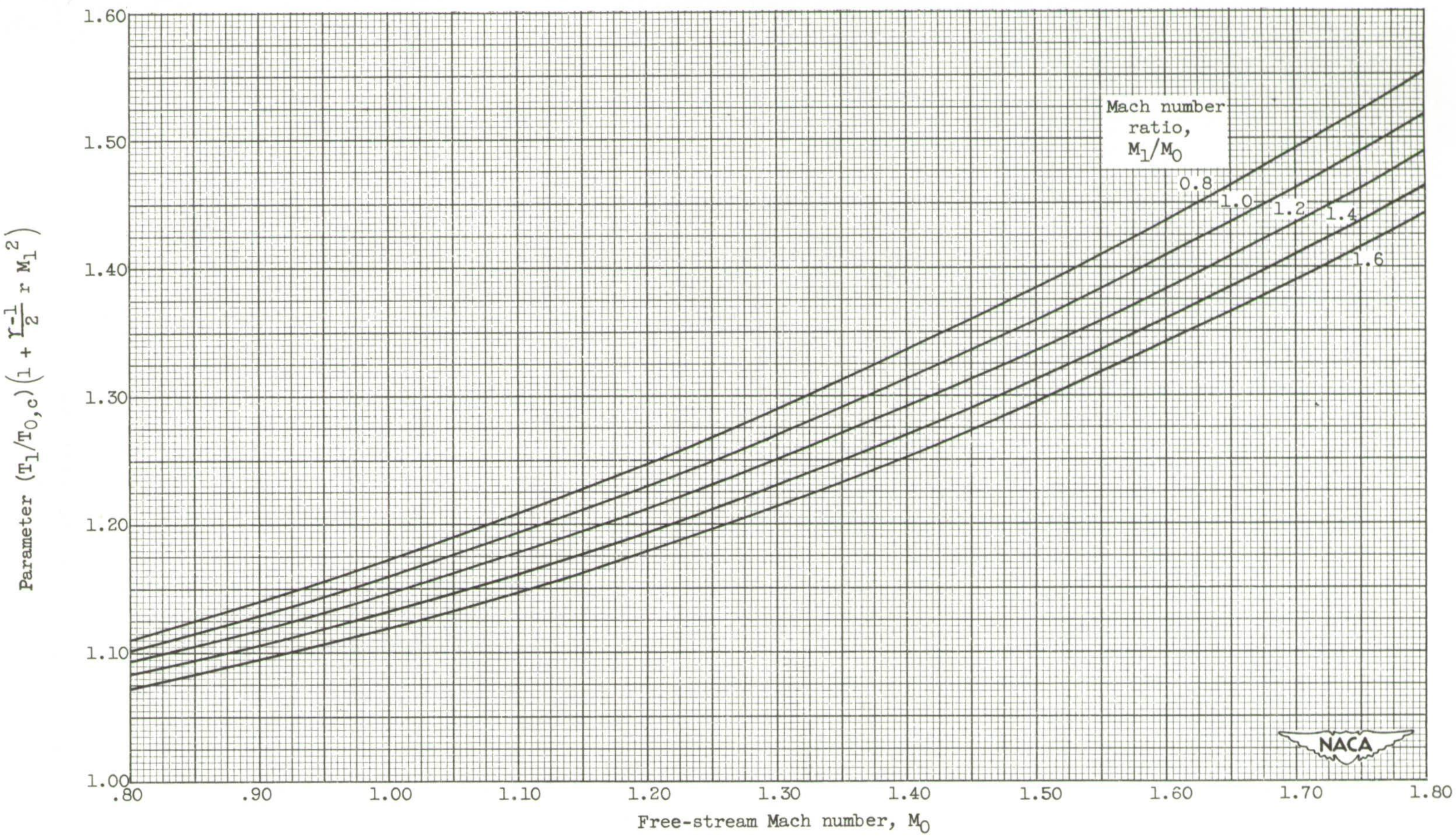
(b) Recovery factor, 0.85.

Figure 3. - Continued. Variation of parameter $(T_1/T_{0,c})\left(1 + \frac{\gamma-1}{2} r M_1^2\right)$ with free-stream Mach number in shock-free flow for various static-pressure ratios (eq. (5)).



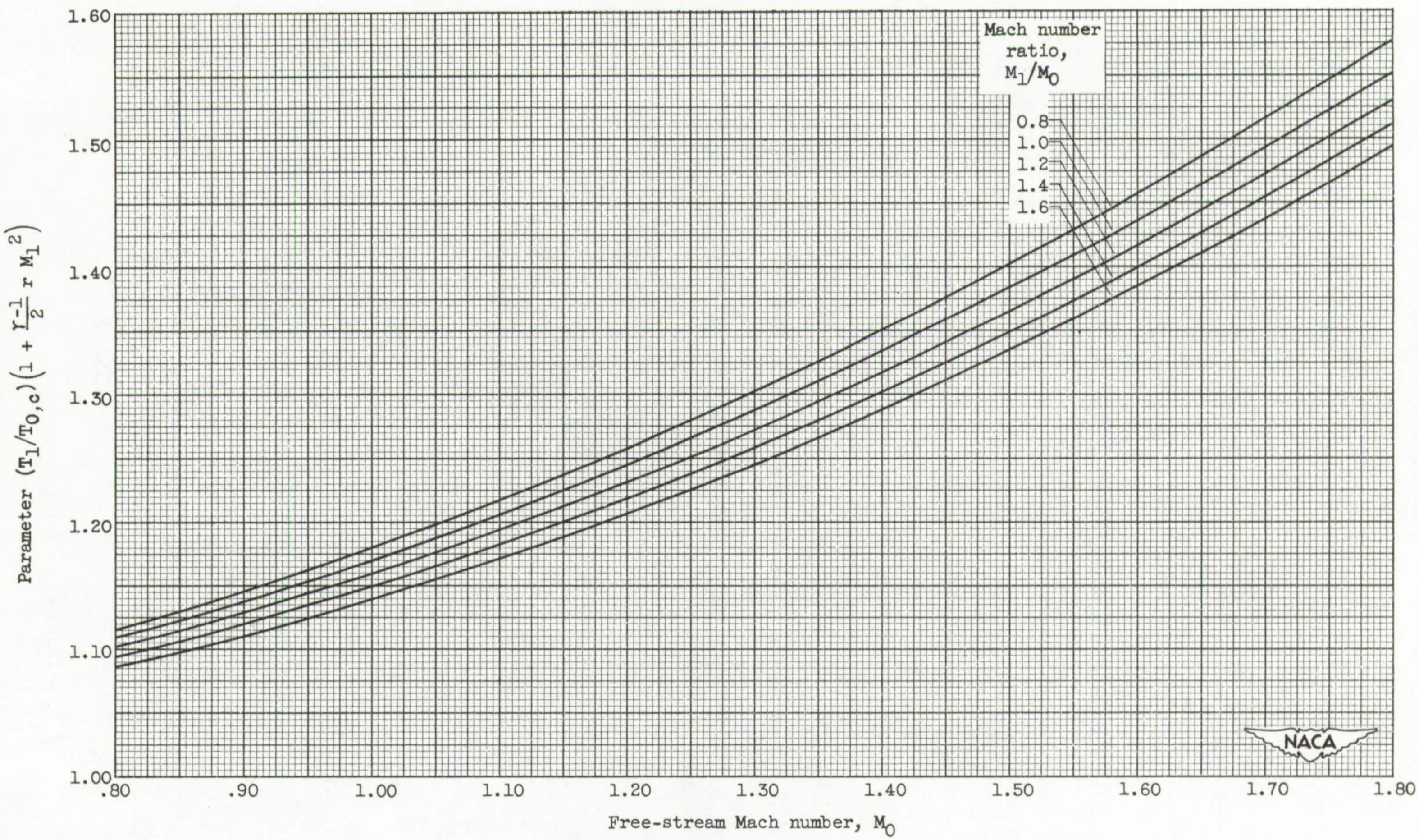
(c) Recovery factor, 0.90.

Figure 3. - Concluded. Variation of parameter $(T_1/T_{0,c})\left(1 + \frac{\gamma-1}{2} r M_1^{\,2}\right)$ with free-stream Mach number in shock-free flow for various static-pressure ratios (eq. (5)).



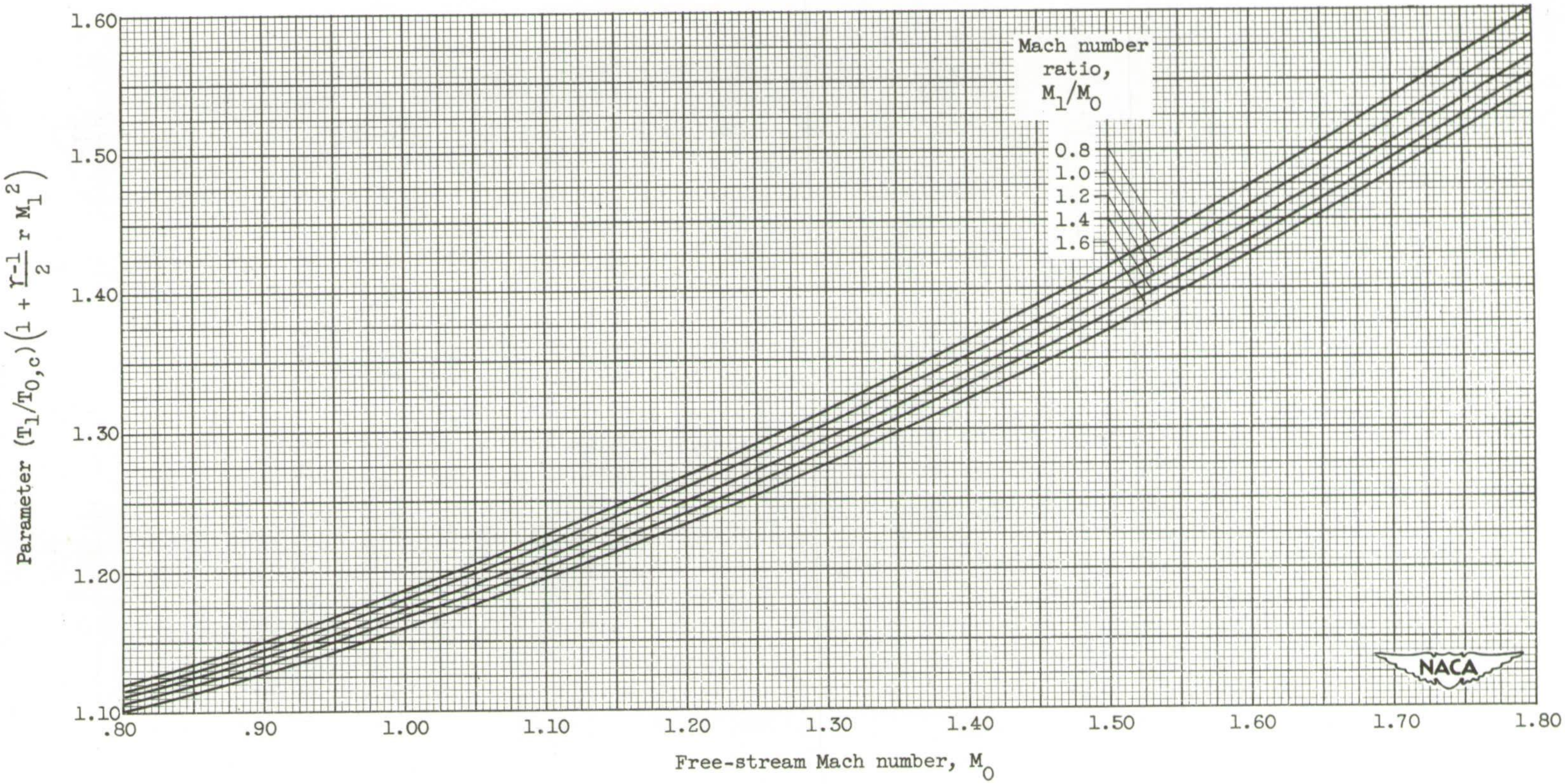
(a) Recovery factor, 0.80.

Figure 4. - Variation of parameter $(T_1/T_{0,c})\left(1 + \frac{\gamma-1}{2} r M_1^2\right)$ with free-stream Mach number in shocked or shock-free flow for various ratios of local to free-stream Mach number M_1/M_0 (eq. (6)).



(b) Recovery factor, 0.85.

Figure 4. - Continued. Variation of parameter $(T_1/T_{0,c})\left(1 + \frac{\gamma-1}{2} r M_1^2\right)$ with free-stream Mach number in shocked or shock-free flow for various ratios of local to free-stream Mach number M_1/M_0 (eq. (6)).



(c) Recovery factor, 0.90.

Figure 4. - Concluded. Variation of parameter $(T_1/T_{0,c})\left(1 + \frac{\gamma-1}{2} r M_1^2\right)$ with free-stream Mach number in shocked or shock-free flow for various ratios of local to free-stream Mach number M_1/M_0 (eq. (6)).

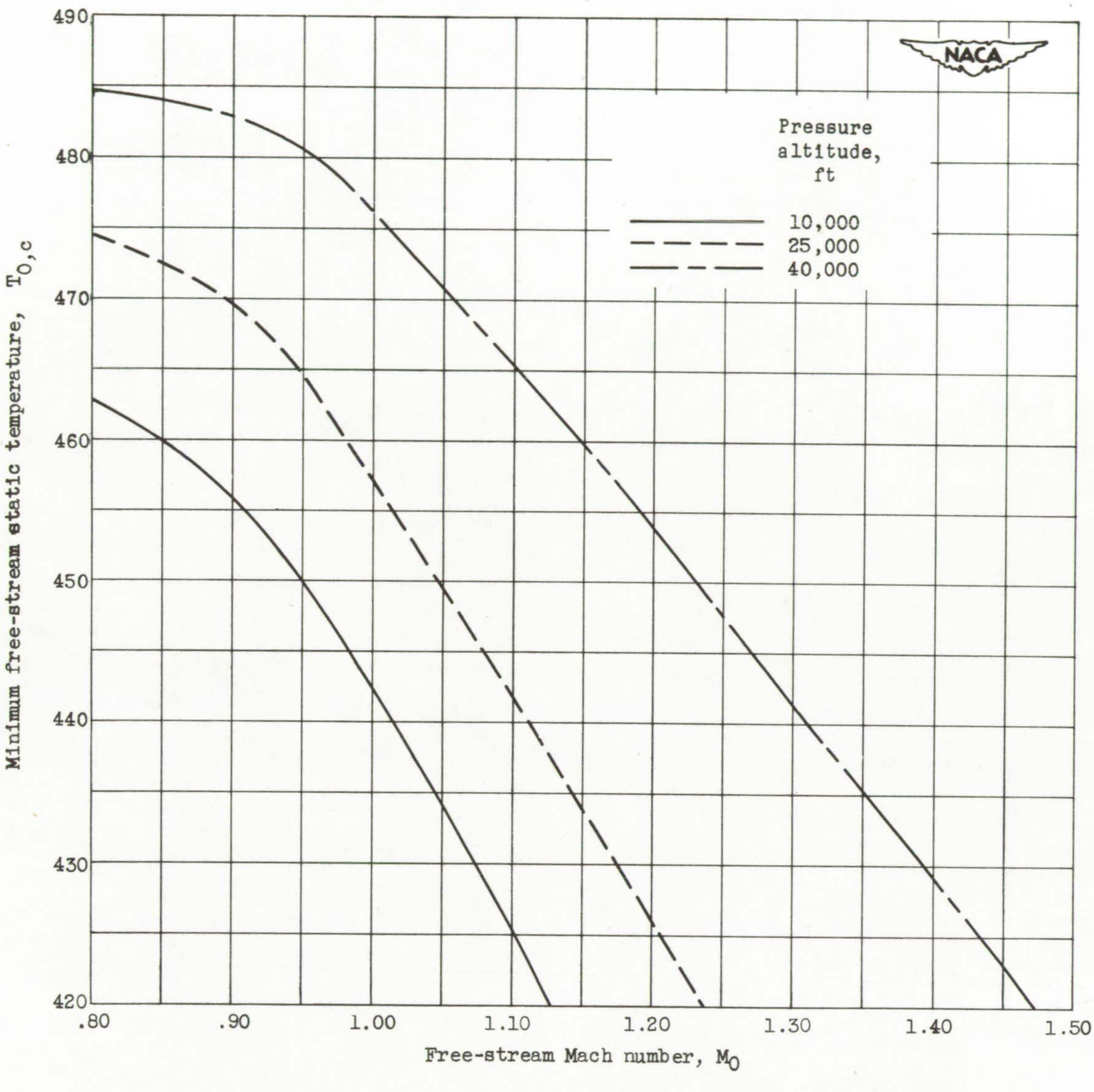


Figure 5. - Variation of minimum free-stream static temperature for ice-free surface as function of free-stream Mach number for station at 50 percent chord of 8.8 percent thick circular arc airfoil.